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ORIENTATION OF THE FIFTEENTH AND SIXTEENTH CENTURY MOSQUES IN THE FORMER YUGOSLAVIA

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Abstract: The paper presents the analysis of the orientation of 60 mosques built in the XV and XVI centuries in the Balkans' region of former Yugoslavia. The mosques have been selected according to their architectural value - mostly the dome mosques that were built by the most renowned builders. Based on the geographic coordinates, the qiblas of all mosques were calculated and the azimuths of their axes measured on orthophotographs. Statistical analysis has shown that the axes of these mosques vary in the horizon sector that is five times wider than the calculated sector of the correct qibla, with a systematic deviation of –10° 15' in relation to the correct qibla. Connections between deviations of the architectural design (dome mosques and other mosques), location and elevation have not been identified. However, a connection between deviations and the time of construction has been identified: deviations from the qibla are smaller in mosques built at a later date. The paper has laid the groundwork for future analysis of the causes of the aforementioned deviations: in the XV and XVI centuries there were no accurate geographic coordinates of locations and the builders were not able to calculate (take over, measure) the exact qibla direction, regardless of the method they applied.

Key words: Mosque orientation, qibla, Yugoslavia, circular statistics, test of uniformity, von Mises distribution

Reasons for studying the orientation of old mosques in the former Yugoslavia

The direction of prayers has been prescribed; Christians face the east or the direction of the rising sun, whereas Muslims face the holy temple in the Kaaba, which is located in the centre of the Grand Mosque in Mecca.

The direction towards Mecca (qibla) determined all the rituals, ceremonies and rites of Muslims wherever they may have found themselves: the direction of the prayers, the orientation of the mosques, maps, graves; all of that had Mecca as their vector. (Podosinov, 1999, p. 328).

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While it was easy to direct the main axis of a church towards the east and even easier towards the rising sun, the direction of a mosque's main axis, the qibla, had to be calculated separately for every location. This enabled medieval Arab scientists to work smoothly and accelerated the development of mathematical geography, astronomy and cartography. Therefore, today, by analizing the relative accuracy of the orientation of old mosques, indirect information on the level of knowledge in a particular period of history can be obtained (Tadić, 1991a). This type of research was especially popular in the last two decades of the twentieth century, but it did not include the central part of the Balkans that had been a part of the Ottoman Empire for five centuries; at that time, it was regarded as a "hopeless place for fieldwork" (King, 1995, p. 267).

In the second half of the twentieth century Yugoslavia (SFRY) occupied a significant part of the Balkan peninsula, where there were around two thousand[2] mosques (Čelić, 1984, p. 716) built over a period of six centuries. In this paper we focus on the orientation of the oldest mosques, constructed in the XV and XVI centuries (Table 1).

Architectual characteristics of old mosques in the former Yugoslavia

The Ottoman conquest of the central Balkans progressed gradually in several stages. Following the Battles of Maritsa and Kosovo (1371, 1389) and the fall of the Serbian Empire, the Ottomans needed the entire XV century to conquer Bosnia (1463) and Herzegovina (1482), penetrate Croatia (1493), reach Belgrade (1521) and cross the Danube and Sava Rivers.With their advancing, at the same time and on the same territory, the Ottoman religious architecture spread with mosque being its principal structure. In the XV and XVI centuries this new architecture existed side by side with the Serbian medieval church architecture, which was "losing its monumentality and stylish character" (Deroko, 1953, p. 298). Frequently, the Ottomans converted monumental churches into mosques, whose original purpose was restored following the liberation from the Ottoman rule. The best known examples are St. Sophia (Ohrid), Mother of God Ljeviška (Prizren) and the Virgin of Gradac (Čačak) (Andrejević, 1976).

Monumental Serbian monasteries consisted mostly of the main church (catholicon) and smaller churches hidden far away from towns, whereas mosques were built in towns, with their towers clearly marking the newly conquered territories.

[2] After the civil war (1992–1995) the number of mosques in the same region has been increasing constantly. It is estimated that today (2015) there are almost 2,400 mosques.

From the architectural point of view, in the former Yugoslavia the most significant are dome mosques in which all three architectural styles are represented: bursa (XIV and the beginning of the XV century), the new Constantinople style (after the fall of Constantinople — the second half of the XV to the first quarter of the XVI century) and the classical Ottoman style (XVI century — “the golden age of the Ottoman architecture”). The greatest number of mosques in the former Yugoslavia belongs to the classical style.

In all dome mosques four basic construction types can be distinguished: single-spaced with a porch covered with three small domes, single-spaced with a porch covered with two small domes, multi-spaced, and mosques with a dome on polygonal bases. “Among the above mentioned types of dome mosques there is not a single one that would have been unknown in the contemporary mosque architecture of the home country.” (Andrejević, 1984, pp. 98–99).

In its basic form, the dome mosque consists of a central cube space for prayers covered by a semicircular dome, with a porch covered with three or two small domes and a lean lanceolate tower with a polygonal base (minaret) located on the right side of the entrance from whose circular balcony (seref) a mosque’s official, the muezzin, while facing Mecca makes the call (ezan) for one of the five daily prayers in Islam. All mosques built in the XV and XVI centuries in the region of the former Yugoslavia had one minaret each and, with a few exceptions, one balcony[3] each (Čelić, 1984, p. 716).

In the middle of the wall, opposite the entrance, is a special niche, mihrab, from which the principal mosque officer, the imam, leads the prayer. The main axis of a mosque, the imaginary line which extends from the entrance towards the mihrab, must be in direction of qibla. For the mosques in the former Yugoslavia, the qibla is, approximately, in the direction of the south-east (Tadić, 1991b).

This is the prescribed (theoretical) situation, but in reality there are frequent deviations in both the old and the new mosques in all parts of the Islamic world, from central Asia to Andalusia (Barmore, 1985; Bonine, 1990, 2008): these deviations among old mosques in the former Yugoslavia are clearly visible on the satellite imagery and orthophotos.

[3] Following the disintegration of Yugoslavia numerous mosques have been erected without respecting the building traditions, among them there are many with two minarets and two or even three balconies.

Criteria for the selection of old mosques in the former Yugoslavia

For this orientation analysis, 60 mosques have been selected (Table 1) from 33 towns in the former Yugoslavia. They were built or at least designed (later reconstructed, demolished, renewed or rebuilt) in the XV century (13 mosques) and the XVI century (47 mosques). Today, they are located in five newly formed states: Macedonia (14), Serbia (15), Montenegro (1), Bosnia-Herzegovina (28), and Croatia (2); the largest number is from the following two cities: Sarajevo (10) and Skopje (5). Two clusters of mosques are clearly separated (Figure 1): mosques built in the region conquered by the Ottomans, mainly by the end of the XV century, which belong to the south-east (SE) group (east from the Meridian 20° E) and the remaining mosques that belong to the north-west (NW) group (see Table 1).

The starting point for the selection were the lists already compiled by art/architectural historians (Andrejević, 1984; Redžić, 1983; Tihić, 1979) and the list of immovable cultural heritage created by the appropriate governmental institutions for the protection of cultural monuments. Almost all of the selected mosques are officially valued as cultural monuments of exceptional or great importance; two mosques converted to churches and four partially or entirely demolished mosques are the exception. In Table 1, the former are labeled with a circular arrow (↺), and the latter with a star (*), in front of their ID numbers.

The Ottomans destroyed and burnt many churches in retaliation, especially in the XVII century during the crisis of the Ottoman Empire. In retaliation, many mosques were destroyed during the wars of liberation of the Balkan nations. The mutual/three-sided settling of old accounts by demolishing temples continued during the Second World War, and then during the armed conflicts following the disintegration of Yugoslavia, when numerous mosques were completely destroyed or damaged. After the war, the mosques were reconstructed on the same foundations; in Table 1 they have been labeled with a vertical arrow (↑), in front of the ID number.

In the second column in Table 1 the names of the mosques have been entered. Unlike churches, the mosques carry the names of the donors (wakifs) from the ranks of rulers and nobility. Besides the official names, some mosques are known by their substitute/picturesque names (Kuršumlija — covered with lead, Aladža — multicoloured, Čaršijska — town, etc.).

Out of the 60 analyzed mosques, two-thirds are with dome, the most valuable from the architectural standpoint (the work of the best builders), while the remaining mosques are mostly with a hipped roof (see the last column in Table 1).

Location, elevation, geographic coordinates and the year of construction have been indicated for each mosque. When the years of construction are not known precisely, rounding has been used for statistical analysis, for example: first half of the XVI century = 1525, mid-sixteenth century = 1550, second half of the XVI century = 1575).

Table 1. XV and XVI century mosques in the former Yugoslavia

ID	Mosque City, Elevation	Year	Lat. Long.	Qibla_calc Qibla_real	Qibla_dif	Dome Group
1	Sultan-Murata II Skoplje, 256	1436	42.00 21.44	312.6 295.0	–17.6	No SE
2	Ishak-begova Skoplje, 255	1438	42.00 21.44	312.6 306.5	–6.1	Yes SE
3	Šarena Tetovo, 472	1438	42.01 20.97	313.4 301.5	–11.9	No SE
↑4	Turhan Emin-begova Ustikolina, 405	1448	43.58 18.79	315.0 305.5	–9.5	No NW
5	Isa-begova Skoplje, 255	1460	42.00 21.44	312.6 301.0	–11.6	Yes SE
6	Taš Priština, 602	1461	42.67 21.17	312.1 307.5	–4.6	Yes SE
7	Al Fatih Priština, 603	1461	42.67 21.17	312.1 308.0	–4.1	Yes SE
8	Careva Sarajevo, 550	1462	43.86 18.43	315.2 294.0	–21.2	Yes NW
9	Hadži-Durgut Ohrid, 703	1466	41.12 20.81	315.0 301.0	–14.0	No SE
10	Labska Priština, 598	1470	42.67 21.16	312.1 303.0	–9.1	Yes SE
*11	Carši Prilep, 655	1475	41.35 21.56	313.3 295.5	–17.8	Yes SE
12	Bajrakli Peć, 511	1475	42.66 20.29	313.7 311.5	–2.2	Yes SE
13	Mustafa-pašina Skoplje, 271	1492	42.00 21.44	312.6 301.5	–11.1	Yes SE
14	Jahja-pašina Skoplje, 258	1504	42.01 21.44	312.6 295.5	–17.1	No SE
15	Isak-begova Bitola, 616	1508	41.03 21.33	314.2 316.0	1.8	Yes SE
16	Suzi Čelebijina Prizren, 404	1513	42.21 20.73	313.5 305.0	–8.5	No SE
17	Ahmed-begova Novi Pazar, 493	1516	43.14 20.52	312.6 295.0	–17.6	No NW
*18	Bali-begova Niš, 205	1516	43.33 21.89	310.0 298.5	–11.5	Yes SE
↑19	Sultana Selima Stolac, 60	1519	43.08 17.96	317.0 311.0	–6.0	No NW
↑20	Sultan Sulejmanova Blagaj, 59	1519	43.26 17.89	316.9 310.0	–6.9	Yes NW

21	Tatar Sinan-begova Kumanovo, 333	1520	42.14 21.71	311.9 292.0	–19.9	Yes SE
↺22	Halila hodze Drniš, 328	1522	43.86 16.15	318.8 301.5	–17.3	No NW
23	Husamedin-pašina Štip, 310	1525	41.74 22.20	311.6 301.0	–10.6	Yes SE
24	Čekrči Muslihidinova Sarajevo, 555	1526	43.86 18.43	315.2 306.5	–8.7	Yes NW
25	Altun-Alem Novi Pazar, 500	1528	43.14 20.52	312.6 296.0	–16.6	Yes NW
26	Arap Novi Pazar, 498	1528	43.14 20.52	312.6 291.0	–21.6	No NW
27	Durak Hadži-ahmetova Sarajevo, 550	1528	43.86 18.43	315.2 318.5	3.3	Yes NW
28	Kodža-kadi Bitola, 621	1529	41.03 21.33	314.2 316.0	1.8	No SE
29	Gazi Husref-begova Sarajevo, 552	1530	43.86 18.43	315.2 289.5	–25.7	Yes NW
30	Magribija Sarajevo, 545	1538	43.86 18.41	315.2 312.5	–2.7	No NW
31	Vekil-Harčova Sarajevo, 553	1541	43.86 18.43	315.2 301.5	–13.7	No NW
32	Hadži Bali-begova Kladanj, 566	1544	44.22 18.69	314.3 294.0	–20.2	Yes NW
33	Divan-katiba Hajdar Sarajevo, 647	1545	43.86 18.44	315.2 310.5	–4.7	No NW
↑34	Aladža Foča, 397	1549	43.51 18.78	315.1 307.5	–7.6	Yes NW
35	Jeni Travnik, 538	1549	44.23 17.67	315.9 322.0	6.1	Yes NW
36	Gazi Ali-begova Vučitrn, 518	1550	42.82 20.96	312.3 296.5	–15.8	No SE
*37	Muslihudina Abdulganija Mažići, 936	1550	42.94 20.95	312.2 304.5	–7.7	Yes SE
38	Nezir-agina Mostar, 57	1550	43.34 17.81	316.9 305.5	–11.4	No NW
↺39	Ibrahim-pašina Djakovo, 110	1550	45.31 18.41	313.4 306.0	–7.4	Yes NW
40	Čejvan-čehajina Mostar, 59	1552	43.34 17.82	316.9 303.5	–13.4	No NW
41	Karadžoz-begova Mostar, 63	1557	43.34 17.81	316.9 298.0	–18.9	Yes NW
42	Kadi Mahmud Efendi Bitola, 616	1558	41.03 21.33	314.2 319.0	4.8	Yes SE
43	Bor Novi Pazar, 497	1560	43.14 20.51	312.7 313.5	0.8	No NW
44	Ali-pašina Sarajevo, 544	1560	43.86 18.41	315.2 309.5	–5.7	Yes NW
45	Jusuf-pašina Maglaj, 185	1560	44.55 18.10	314.9 277.5	–37.4	Yes NW

↑46	Hajdar-kadijina Bitola, 612	1561	41.03 21.34	314.1 302.5	–11.6	Yes SE
47	Ferhad-begova Sarajevo, 533	1561	43.86 18.43	315.2 315.0	–0.2	Yes NW
↑48	Hadži-Alijina Pocitelj, 42	1562	43.13 17.73	317.3 308.0	–9.3	Yes NW
49	Hadži-Ahmetova Livno, 785	1562	43.83 17.01	317.5 307.5	–10.0	Yes NW
50	Nasuh-age Vučijakovića Mostar, 67	1564	43.34 17.82	316.9 296.0	–20.9	Yes NW
51	Coban H. Hasan-vojvode Sarajevo, 546	1565	43.86 18.42	315.2 296.0	–19.2	No NW
*52	Sinan-begova Čajniče, 806	1570	43.56 19.07	314.5 308.5	–6.0	Yes NW
53	Gazi Mehmed-pašina Prizren, 418	1573	42.21 20.74	313.5 323.5	10.0	Yes SE
54	Husein-pašina Pljevlja, 772	1573	43.36 19.36	314.3 313.0	–1.3	Yes NW
55	Balaguša Livno, 801	1575	43.83 17.01	317.5 296.0	–21.5	Yes NW
56	Lala-pašina Livno, 750	1577	43.83 17.01	317.5 305.5	–12.0	Yes NW
↑57	Ferhadija Banjaluka, 165	1579	44.77 17.19	316.0 315.5	–0.5	Yes NW
58	Hasan-agina Rogovo, 318	1580	42.33 20.58	313.6 307.0	–6.6	Yes SE
59	Hadum-begova Đjakovica, 362	1595	42.38 20.43	313.8 289.0	–24.8	Yes SE
60	Sinan-pašina Kačanik, 480	1596	42.23 21.26	312.6 307.0	–5.6	Yes SE

There are no names of builders in Table 1 as they are unknown for most of the mosques, the same as for majority of medieval Serbian churches.

The qibla of old mosques in the former Yugoslavia

The qibla is expressed in degrees (q) and in the Islamic world is measured from the southern point of the horizon, in the positive direction. It is calculated on the basis of the geographic coordinates of the given location T (φ, λ) and Mecca (φ_M, λ_M), using the formula (Roegel, 2008, p. 6),

$$\cot q = \frac{\sin\varphi\cdot\cos\Delta\lambda - \cos\varphi\cdot\tan\varphi_M}{\sin\Delta\lambda} \quad (1)$$

In accordance with the formula and on the basis of geographic coordinates of the mosques (φ, λ; GWS84) which are determined in geoportals and on *Google Earth,* the qiblas for all analyzed mosques have been calculated, and the values given in Table 1 (Qibla-calc). Today, it is easy to check if the main axes of the mosques are in the correct qibla by measuring the angles, without the need to go to the site itself.

The azimuths of old mosques in the former Yugoslavia

Out of the analyzed mosques in the former Yugoslavia, the azimuths of the axes of the Sarajevo mosques and the axes of the Ferhadija mosque in Banja Luka were measured on the appropriate survey sheets with the 1:500 and 1:1000 scale, whereas the azimuths of the axes of other mosques were measured using digital orthoimages which are now widely available on the geoportals of the successor states of the former Yugoslavia.

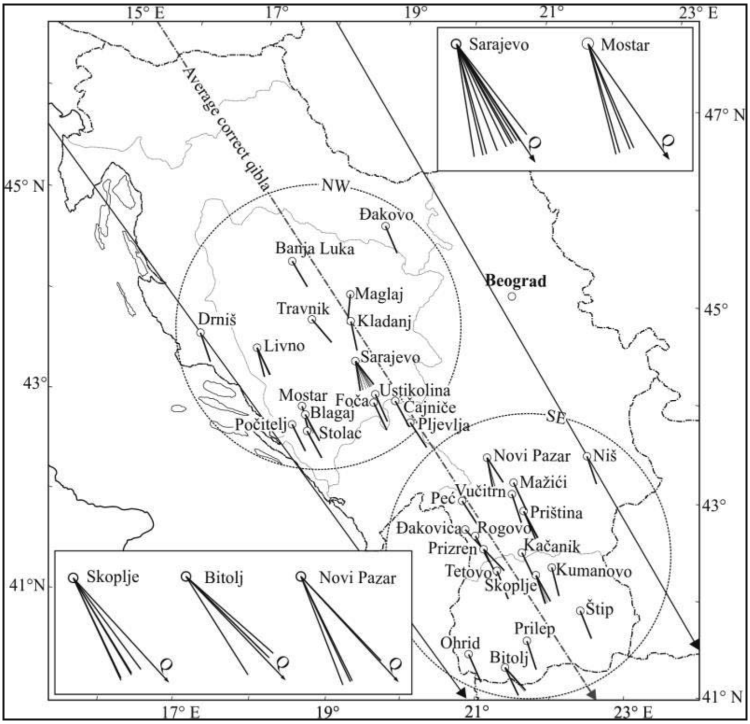

Figure 1. The direction of the main axes of the selected old mosques in the former Yugoslavia

Due to poor quality of the images, i.e. lack of sharpness of the object' edges, the azimuths could not be measured with the accuracy greater than ±1° on some of these geoportals, which is, nevertheless, sufficient for the intended analysis. The results of azimuth measurements (Qibla_real) are presented in Table 1 and shown on the map (Figure 1).

The map clearly shows that the mosque axes were not directed towards the same point, that is, that they were removed from the correct qibla to a varying extent (Table 1: Qibla_diff). These deviations will be the subject of the statistical analysis in the next section.

Statistical analysis of the orientation of the mosques

Research questions

The main objective of this section is to identify whether there are any patterns in the way the XV and XVI century mosques in the former Yugoslavia are oriented. More specifically, the data gathered for this paper were used to address the following four research questions:

- What are the stylized facts about the orientations of the XV and XVI century mosques in the former Yugoslavia?
- Does the orientation of the mosques point to the correct direction, i.e. toward Mecca (correct qibla)?
- Is there any impact on the orientation of the mosques in relation to their construction – if they were built with a dome, built in the XV or XVI century or located in north-west or south-east regions of the former Yugoslavia?
- Is there any association between the elevation at which a mosque was built and the year when it was built on one side and the deviation of its qibla from the correct one on the other side?

Data description and summary statistics

Table 2 lists all the variables we have used with data description, their domains and measurement units.

Table 2. Description of data used

Acronym	Description, domain and measurement unit
Year	Construction starting year (estimated)
Qibla_calc	Correct qibla (degrees), i.e. correct direction to Mecca
Qibla_real	Real qibla (degrees), i.e. actual qibla direction
Qibla_diff	Difference between actual qibla direction and correct qibla (degrees)
Dome	Mosque built with dome (Yes/No)
Group	Mosque built in north-west or south-east region of the former Yugoslavia (NW/SE)
Elevation	Elevation (meters)
Century	Mosque built in XV or XVI century (XV/XVI)

The calculations in this paper were performed with R package “circular” (Agostinelli & Lund, 2015).

To address the first two research questions we have used summary statistics and the rose diagram as a graphical display of circular data.

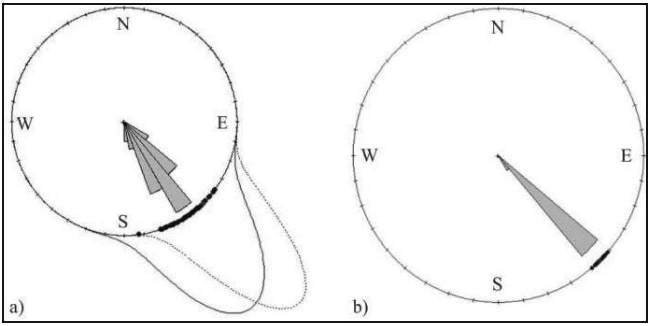

Figure 2. (a) Rose diagram of Qibla_real for 60 mosques in former Yugoslavia with the kernel density estimation using von Mises smoothing kernel (solid line — distribution of Qibla_real, dotted line — distribution of Qibla_calc) (b) Rose diagram of correct qiblas (red dot on both diagrams represents mean of all Qibla_calc)

We have overlaid kernel density estimate based on von Mises distribution on the rose diagram. The rose diagram and the non-uniform shape of von Mises distribution show clearly that there were attempts to achieve religiously-correct alignment (i.e. orientation to Mecca). Although for some mosques the attempts to achieve the alignment were met successfully, the overall result is a systematic underestimation of the correct orientation to Mecca, which is illustrated with two non-overlapping von Mises distributions in Figure 2(a).

On average, actual qiblas show systematic deviation from the correct direction of -100 15’. There are small differences between mean qibla directions among mosques built with or without a dome, those from north-west and south-east regions of the former Yugoslavia and mosques built in the XV or XVI centuries (see Table 1). For all mosques and for each of these three categorical variables mean direction values are smaller than the corresponding median direction values. This is an indication of left skewness in the distribution of actual qiblas. Since the problem with outliers in circular data is less prominent than in linear data we have continued using mean values instead of median.

Mean resultant length is a measure of data concentration. Mean resultant length values over 0.98 in Table 3 show that overall and for three categorical variables actual qiblas are highly concentrated around particular direction, but not necessarily around the correct direction as we explained above.

Table 3. Summary statistics of mosque qiblas in former Yugoslavia

Statistic	All	Dome		Group		Century	
		Yes	No	NW	SE	XV	XVI
Number of mosques	60	41	19	34	26	13	47
Mean direction (degrees)	304.1	304.6	303.0	304.2	304.0	302.4	304.6
Median direction (degrees)	305.4	306.5	301.5	305.8	302.8	301.5	305.5
Mean deviation (degrees)	-10.3	-9.7	-11.6	-11.3	-8.9	-10.8	-10.1
Mean resultant length (rho) (radians)	0.988	0.986	0.993	0.987	0.990	0.996	0.986
Circular standard deviation (radians)	0.154	0.166	0.162	0.162	0.142	0.090	0.166
Concentration parameter (kappa)	42.8	36.9	67.0	38.6	49.9	124.0	36.7

Source of data: Authors calculation

Finally, small values of the circular standard deviation, as a measure of variation in the data, indicate little variations in the orientation of the mosques. The same message is conveyed by the concentration parameter kappa which is the estimated concentration parameter of the von Mises distribution.

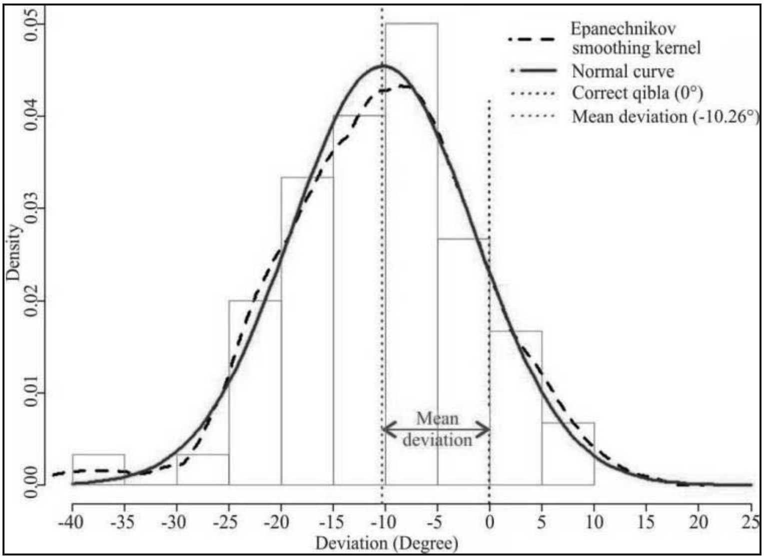

Figure 3. Histogram of deviations of the real from correct qibla with kernel density estimation (Epanechnikov smoothing kernel) and normal curve

Figure 3 displays a histogram of deviations of the real from correct qiblas with the kernel density estimate. Left skewness of the distribution of deviations is prominent and only a few mosques (7 out of 60) have overestimated correct orientation to Mecca. Although both the kernel density and the normal curve approximate this distribution quite well, the systematic deviation of actual qiblas from correct ones is obvious as indicated by “Mean deviation” in Figure 3.

Testing hypothesis about the orientation of the mosques

To test the hypothesis about uniformity, when all values around the circle, i.e. the orientation of the mosques, are equally likely against an alternative hypothesis of one-sidedness, or directedness, when all qiblas are pointing to Mecca, several statistical tests were developed. The results of four tests of uniformity are presented in Table 4.

Table 4. Statistical tests of uniformity of mosque qiblas in former Yugoslavia

Test	All	Dome		Group		Century	
		Yes	No	NW	SE	XV	XVI
Rayleigh's test of uniformity	0.988	0.986	0.993	0.990	0.990	0.987	0.989
Kuiper's test of uniformity	7.04	5.81	4.25	5.31	4.79	3.12	6.42
Rao's spacing test	313.5	307.9	316.1	306.3	311.7	295	311.1
Watson's test for circular uniformity	4.27	2.90	1.43	2.42	1.91	0.74	3.58

Note: The *p*-values for all the test statistics are less than 1%
Source of data: Authors calculation

Since the p-values for all four tests are well below 1% we conclude that the null hypothesis of circular uniformity should be rejected in favor of an alternative hypothesis that 60 mosques in the former Yugoslavia have qiblas oriented in a particular direction. This result is further confirmation of the distribution presented on the rose diagram in Figure 1(a).

Table 5 mosque qiblas in former Yugoslavia

Test	Dome	Group	Century
Two or three sample tests for mean direction			
High-concentration *F*-test	0.41 (0.52)	0.003 (0.95)	0.60 (0.44)
Likelihood ratio test	0.42 (0.51)	0.004 (0.95)	0.61 (0.43)
Two or three sample tests for concentration parameter			
Mardia & Jupp test	1.81 (0.18)	0.43 (0.51)	4.93 (0.03)
Two or three sample tests of angular distance/dispersion			
Wallraff's test	1.12 (0.29)	0.78 (0.38)	4.22 (0.04)

Note: Test statistic (*p*-value)
Source of data: Authors calculation

To test the hypothesis about equality of mean directions, concentration parameters and angular distances between mosque qiblas belonging to one of the two categories of three categorical variables considered (Dome, Group and Century) three tests were used and results are presented in Table 5. These results will be used to address the third research question.

All the test statistics and their p-values, with two exceptions, suggest that we have not been able to reject null hypothesis of equal mean directions of the orientation of the mosques for all three categorical variables (Dome, Group and

Century) and equal concentration parameters and angular distances. The two exceptions mentioned above are results of the Mardia & Jupp test for equal concentration parameters and Wallraff's test of equal angular distances for the Century variable. We conclude that concentration parameters and angular distances in mosques from the XV and XVI centuries are statistically different at the 3% and 4% levels respectively. Therefore, only the century in which the mosque was built may have had impact on their orientation. The facts that a mosque was built with a dome or not and whether it is located in north-west or south-east regions of the former Yugoslavia have no impact on its orientation.

To address the last research question about the association between deviations of qiblas from correct orientation and elevation at what a mosque was built and the year when a mosque was built the circular-linear correlation coefficient was used. This coefficient with statistical test of its significance is presented in Table 6.

Table 6. Johnson-Wehrly-Mardia circular-linear correlation coefficients and test of significance

Variables	Correlation	*F*-test	*p*-value
Qibla_diff, Elevation	0.03	2.03	0.14
Qibla_diff, Year	0.07	4.21	0.02

Source of data: Authors calculation

Both correlation coefficients indicate weak association between elevation and the year when a mosque was built and deviation of qiblas from the correct orientation. However, while correlation coefficient of 0.03 is not statistically significant indicated by large p-value (0.14), association between the year when the mosque was built and deviation of qiblas from the correct orientation is statistically significant at 2% level. This means that a qibla of a mosque that was built later would deviate less on average from the correct qibla.

Appendix: About circular statistics

Here we provide a brief overview of the main circular statistics and von Mises distribution. Complete texts on this subject were written by Gaile and Burt (2008), Jammalamadaka and Sengupta (2001) and Mardia and Jupp (2000). Pewsey, Neuhauser and Ruxton (2013) give an overview of the circular statistics and its application using R package.

The following simple example illustrates the need for circular statistics. Suppose we have two observations 50 and 3550. Standard mean value would be computed as (5+355)/2=1800. This result shows that applying a standard mean to cyclic measurement makes little sense because these two observations are actually clustered around 00. In situations such as this, a more appropriate circular mean would give 00.

Data in circular statistics are represented by a unit length vector: x=[cosθ sinθ]'.

To calculate mean direction and mean resultant vector we find the following two mean values: $\bar{c} = \sum_{i=1}^{n} \cos\theta_i / n$; $\bar{s} = \sum_{i=1}^{n} \sin\theta_i / n$. Then the vector $\bar{r} = [\bar{c}\bar{s}]'$ is the mean resultant vector of n observations, with mean resultant length: $\bar{r} = \sqrt{\bar{c}^2 + \bar{s}^2} \in [0,1]$, and mean direction (for $\bar{r} \neq 0$):

$$\bar{\theta} = \arctan(\bar{s}/\bar{c}) \text{ if } \bar{c} \geq 0$$

$$\bar{\theta} = \arctan(\bar{s}/\bar{c}) + \pi sign(\bar{s}) \text{ if } \bar{c} < 0 \quad (2)$$

The mean resultant length can be used as a measure of distribution concentration. If all the vectors are concentrated in the same direction, as expected with mosque qiblas, then the mean resultant length is equal 1. On the other extreme, when the vectors are uniformly distributed around the unit circle, then the mean resultant length is equal 0, because there is no preferred direction.

Based on the mean resultant length the sample circular variance is defined: $v = 2(1 - \bar{r}) \in [0,2]$. Then the sample standard deviation is defined as: $s = \sqrt{-2\ln\bar{r}}$, which reduces to $\sqrt{v}$ for small *v*.

The von Mises distribution takes the role in circular statistics that is held by the normal distribution in standard linear statistics. As illustrated in Figure 1(a) it is bell-shaped curve like normal distribution, but its tails are truncated.

Its probability density function is given by $f(\theta, \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa\cos(\theta-\mu)}$ where $\kappa \geq 0$ is concentration parameter and $I_0(\kappa)$ is modified Bessel function of the first kind and order 0 defined by $I_0(\kappa) = \int_0^{2\pi} e^{\kappa\cos(\theta)} d\theta$.

Johnson-Wehrly-Mardia circular-linear correlation coefficient

We use a random sample (x1, θ1), (xn, θn), of observations on (X, Θ), where X is a linear and Θ is a circular variables. First, the Pearson correlation coefficients should be calculated:

rxc = r((x1, …, xn), (cos θ1, …, cos θn)), rxs = r((x1, …, xn), (sin θ1, …, sin θn)),

rcs = r((cos θ1, …, cos θn), (sin θ1, …, sin θn)).

The Johnson-Wehrly-Mardia circular-linear correlation coefficient is then given by

$$R_{x\theta}^2 = \frac{r_{xc}^2 + r_{xs}^2 - 2r_{xc}r_{xs}r_{cs}}{1 - r_{cs}^2} \tag{3}$$

This correlation coefficient ranges between zero and one. The greater its value the stronger the association between linear and circular variables.

Conclusion

The qiblas of the 60 analyzed mosques vary in the interval from 309° 58' to 318° 47', with the mean value of 314° 22', whereas the axes of the same mosques vary in a fivefold wider interval from 277° 30 'to 323° 30', with the mean value of 304° 07'. On average, in relation to the correct qibla of the mosque axes they demonstrate a systematic deviation of –10° 15'.

The observed deviations do not mean that the mosque builders on the Balkans in the XV and XVI centuries did not know how to determine and mark the qibla accurately. This claim could be true had there been accurate geographic coordinates of Mecca and the locations of the mosques: there were none and so they were not able to mark the qibla accurately, no matter whether they copied the data from tables, used qibletnams or whether they determined the qibla themselves (regardless of the method known at the time, geometric or trigonometric). This is also true of builders in the entire Islamic world of that period.

Only when, without trying to view the past from contemporary perspective, we decipher the most accurate qibla that the builders in the XV and XVI centuries could have determined on the basis of the available input data and methods, and taking into account all other possible causes, objective and subjective, we can judge the accuracy of the orientation of the mosques, the builders' ability and the level of mathematical and geographical knowledge at the time. This paper has laid the groundwork for future analyses of this kind.

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